7/7

9.0

Mechanica Midtoets 1 3-12-09

1) a) $\vec{a} = (6,3,-2)$ $\vec{b} = (1,2,2)$

$\vec{a} \cdot \vec{b} = 6 \cdot 1 + 3 \cdot 2 + -2 \cdot 2 = 6 + 6 - 4 = 8$

b) $\vec{a} \times \vec{b} = (a_y b_z - b_y a_z, a_z b_x - a_x b_z, a_x b_y - a_y b_x) = (3 \cdot 2 - -2 \cdot 2, -2 \cdot 1 - 6 \cdot 2, 6 \cdot 2 - 3 \cdot 2) = (10, -14, 9)$

c) $\vec{a} \cdot \vec{b} = ab \cos\theta = 8$

$a = \sqrt{6^2 + 3^2 + (-2)^2} = \sqrt{36+9+4} = \sqrt{49} = 7$

$b = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$7 \cdot 3 \cdot \cos\theta = 8$

$\cos\theta = \frac{8}{21}$

$\theta = \cos^{-1}\left(\frac{8}{21}\right)$

Mechanica Middoets 1 3-12-'09

9

2) a) $M\ddot{x} = -At^2$

$M\frac{dv}{dt} = -At^2$

$dv = \frac{-At^2}{M}\,dt$

$\int dv = \frac{-A}{M}\int t^2\,dt$

$v = -\frac{A}{M}\left(\frac{1}{3}t^3 + C\right)$

$v(t) = -\frac{A}{M}\cdot\frac{1}{3}t^3 + C\frac{A}{M}$

$v(t=0) = C\cdot\frac{A}{M} = v_0$

$v(t) = v_0 - \frac{A}{3M}t^3$

9

$\frac{dx}{dt} = v_0 - \frac{A}{3M}t^3$

$dx = v_0 - \frac{A}{3M}t^3\,dt$

$\int dx = \int v_0 - \frac{A}{3M}t^3\,dt$

$x(t) = v_0 t - \frac{A}{3M}\cdot\frac{1}{4}t^4 + C$

$= v_0 t - \frac{A}{12M}t^4 + C$

$x(t=0) = C = x_0$

$x(t) = v_0 t - \frac{A}{12M}t^4 + x_0$

9

b) $v(t) = v_0 - \frac{A}{3M}t^3 = 4 - \frac{6}{12}t^3 = 4 - \frac{1}{2}t^3$

als t klein is ($\frac{1}{2}t^3 < 4$) dan is $v > 0$

als t groter wordt wordt $v < 0$

} → de snelheid keert om van richting

c) van richting omkeren: $v(t) = 0$

$4 - \frac{1}{2}t^3 = 0$

$4 = \frac{1}{2}t^3$

$t^5 = 8$

$t = 2 = t_1$

$x(t_1) = v_0 t_1 - \frac{A}{12M}t_1^4 + x_0 = 4\cdot 2 - \frac{6}{12\cdot 4}\cdot 2^4 + 0 = 8 - \frac{1}{8}\cdot 16 = 8 - 2 = 6$ m

1

(8)

Mechanica midtaets, 3-12-'09

(3) a) $V = -Axe^{-x}$

$\frac{dV(x)}{dx} = -Ae^{-x} - Axe^{-x}\cdot(-1) = Axe^{-x} - Ae^{-x} = Ae^{-x}(x-1)$

$F(x) = -\frac{dV(x)}{dx} = -Ae^{-x}(x-1)$

3 $F(x) = 0$

$\Rightarrow -Ae^{-x}(x-1) = 0$

$x - 1 = 0$

$x = 1$

b) ~~$V(x) = V(x_0) + \frac{1}{1!}V'(x_0) + \frac{1}{2!}V''(x_0)$~~

~~$V(x_0) = Ax_0e^{-x_0}$~~

$V'(x) = \frac{dV}{dx} = Ae^{-x}(x-1)$

~~$V'(x_0) = Ae^{-x_0}(x_0-1) = A$~~

$V''(x) = Ae^{-x} + Axe^{-x}\cdot(-1) - Ae^{-x}\cdot(-1) = Ae^{-x} - Axe^{-x} + Ae^{-x} = 2Ae^{-x} - Axe^{-x} = Ae^{-x}(2-x)$

~~$V''(x_0) = Ae^{-x_0}(2-x_0) = 2A$~~

~~$V(x) = Ax_0e^{-x_0} + A(x-x_0) + \frac{1}{2}2A(x-x_0)^2 = A$~~

$V(x) = V(1+\varepsilon) = V(1+(x-1)) = V(1) + \frac{x-1}{1!}V'(1) + \frac{(x-1)^2}{2!}V''(1)$

$V(1) = -A\cdot 1e^{-1} = -A\cdot e^{-1} = -\frac{A}{e}$ 8

4 $V'(1) = A\cdot e^{-1}(1-1) = A\cdot e^{-1}(0) = 0$

$V''(1) = A\cdot e^{-1}(2-1) = A\cdot e^{-1}\cdot 1 = \frac{A}{e}$ 8

$V(x) = -\frac{A}{e} + \frac{x-1}{1}\cdot 0 + \frac{(x-1)^2}{2}\cdot\frac{A}{e} = -\frac{A}{e} + \frac{x^2-2x+1}{2}\frac{A}{e} = \frac{A}{e}\left(\frac{1}{2}x^2 - x + \frac{1}{2} - 1\right) =$

$\frac{A}{e}\left(\frac{1}{2}x^2 - x - \frac{1}{2}\right) = \frac{1}{2}\frac{A}{e}(x^2 - 2x - 1)$

1 c) $\frac{dV(x)}{dx} = \frac{1}{2}\frac{A}{e}(2x-2) = \frac{A}{e}(x-1)$ $F(x) = -\frac{dV(x)}{dx} = -\frac{A}{e}(x-1) = \frac{A}{e}(1-x)$ 9

d) ???

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